Enrollment No: ______________________ **Exam Seat No:**______________________

C.U.SHAH UNIVERSITY

Winter Examination-2020

Subject Name: Linear Algebra

Subject Code: 5SC01LIA1 **Branch: M.Sc. (Mathematics)**

Semester: 1 **Date: 08/03/2021** **Time: 11:00 To 02:00** **Marks: 70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1 **Attempt the following questions** **(07)**

a) If A and B are finite dimensional subspaces of a vector space V, then $A+B$ is finite dimensional and $\dim(A+B) = \dim A + \dim B - \dim(A \cap B)$. **(02)**

b) Prove that $L(S)$ is subspace of V. **(02)**

c) Let V be a finite dimensional vector space over F and $S, T \in A(V)$.show that $rank(ST) \leq rank(T)$. **(02)**

d) Define: Minimal Polynomial for T. **(01)**

Q-2 **Attempt all questions** **(14)**

a) Let V be a vector space over F then prove that $V$is isomorphic to a subspace of $\hat{\hat{V}}$. If V is finite dimensional then $V \cong \hat{\hat{V}}$. **(07)**

b) Let V be a finite dimensional vector space over F and W be subspace of V. Show that W is finite dimensional,$\dim W \leq \dim V$ and $\dim(V/W) = \dim V - \dim W$. **(07)**

OR

Q-2 **Attempt all questions** **(14)**

a) Let V and W be vector space over F of dimension m and n respectively. Then prove that $HOM(V, W)$ is of dimension mnover$F$. **(06)**

b) Prove that $W^{00} = W$ **(05)**

c) If $v_1, v_2, \dots \dots \dots . v_n$ are in V then either they are linearly independent or some v_k is a linear combination of preceding one's $v_1, v_2, \dots \dots \dots . v_{k-1}$. **(03)**

Q-3 **Attempt all questions** **(14)**

a) If $\mathcal{A}$ is an algebra over F with unit element then prove that $\mathcal{A}$ is isomorphic to a subalgebra of $A(V)$ for some vector space V over F. **(05)**

b) If V is finite dimensional over F, then prove that $T \in A(V)$ is regular if and only if T maps V on to V. **(05)**

c) Let V be finite dimensional over F and $T \in A(V)$ show that the number of characteristic root of T is atmost n^2. **(04)**

OR

Q-3 **Attempt all questions** **(14)**

a) If V is finite dimensional over F, then prove that $T \in A(V)$ is invertible if and only if the constant term in the minimal polynomial for T is nonzero. **(05)**

b) If V is finite dimensional over F,and let $S, T \in A(V)$ and S be regular , then prove that $\lambda \in F$ is chatracteristic root of T if and only if it is a characteristic root of $S^{-1}TS$. **(05)**

c) Let V be a finite dimensional vector space over F. If $T \in A(V)$ is right invertible then T is invertible. **(04)**

SECTION – II

Q-4 **Attempt the following questions** **(07)**

a) Let $A, B \in M_n(F)$,show that $AB - BA \neq I$. **(02)**

b) If $A \in M_n(F)$ is regular then $\det(A) \neq 0$ and $\det(A^{-1}) = \frac{1}{\det A}$. **(02)**

c) Find the inertia of quadraticequation $2x_1x_2 + 2x_1x_3 = 0$. **(02)**

d) Define: Basic Jordan block. **(01)**

Q-5 **Attempt all questions** **(14)**

a) Let V be a finite dimensional vector space over F and $T \in A(V)$ be nilpotent. Then show that the invariants of T are unique. **(07)**

b) Let V be a finite dimensional vector space over F and $T \in A(V)$. If all the characteristic roots of T are in F then there is a basis of with respect to which the matrix of T is (upper) triangular. **(07)**

OR

Q-5 **Attempt all questions** **(14)**

a) Let V be a finite dimensional vector space over F and $T \in A(V)$ and W be subspace of V invariant under. Then T induce a map $\bar{T}: V/W \to V/W$ defined by $\bar{T}(v + W) = Tv + W$ show that $\bar{T} \in A(V/W)$. Further $\bar{T}$ satisfies every polynomial satisfies by T. If $p_1(x)$ and $p(x)$ are minimal polynomial for $\bar{T}$ and T respectively then show that $p_1(x)/p(x)$. **(07)**

b) Two nilpotent linear transformations are similar if and only if they have the same invariants. **(07)**

Q-6 **Attempt all questions** **(14)**

a) Let $A, B \in M_n(F)$,show that $\det(AB) = \det A \cdot \det B$. **(05)**

b) Identify the surface given by $11x^2 + 6xy + 19y^2 = 80$. Also convert it to the standard form by finding the orthogonal matrix P. **(05)**

c) Interchanging two rows of matrix changes the sign of its determinant. **(04)**

OR

Q-6 **Attempt all questions** **(14)**

a) State and prove Cramer's rule. **(05)**

b) Let $f: \boldsymbol{R}^n \times \boldsymbol{R}^n \to \boldsymbol{R}$ be a map. Then f is bilinear if and only if there exist $\alpha_{ij} \in \boldsymbol{R}$, $\boldsymbol{1 \leq i, j \leq n}$with $\alpha_{ij} = \alpha_{ji}$ such that $f(x, y) = \sum_{i,j=1}^{n} \alpha_{ij} x_i y_j$. **(05)**

c) Let F be a field of characteristic 0 and V be a vector space over F. If $S, T \in A(V)$ such that $ST - TS$ commutes with S then show that $ST - TS$ is nilpotent. **(04)**